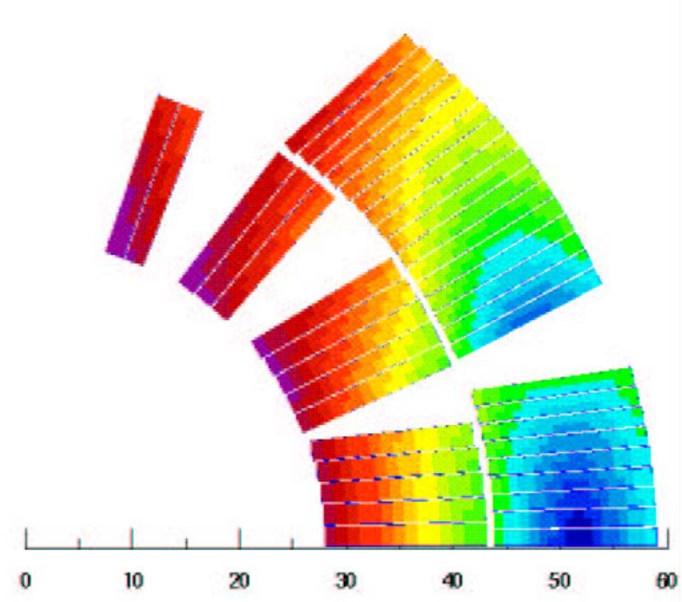
Lecture 2: Magnets & training, plus fine filaments

Magnets

- magnetic fields above 2 Tesla
- coil shapes for solenoids, dipoles and quadrupoles
- engineering current density
- load lines



Degradation & Training

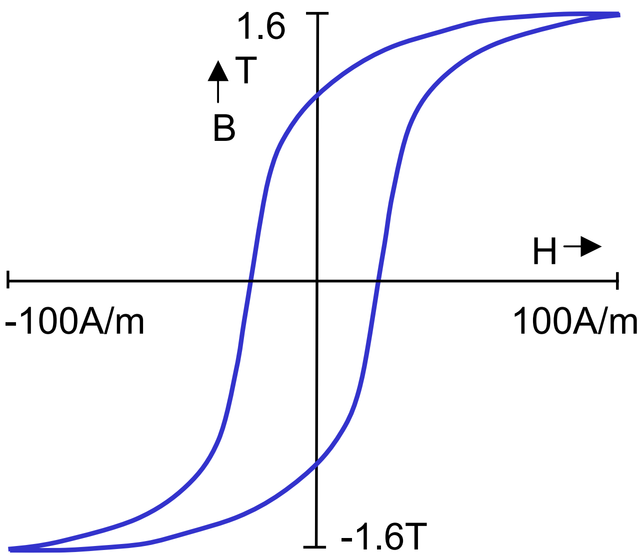
- causes of training - release of energy within the magnet
- minimum propagating zones MPZ and minimum quench energy MQE

Fine filaments

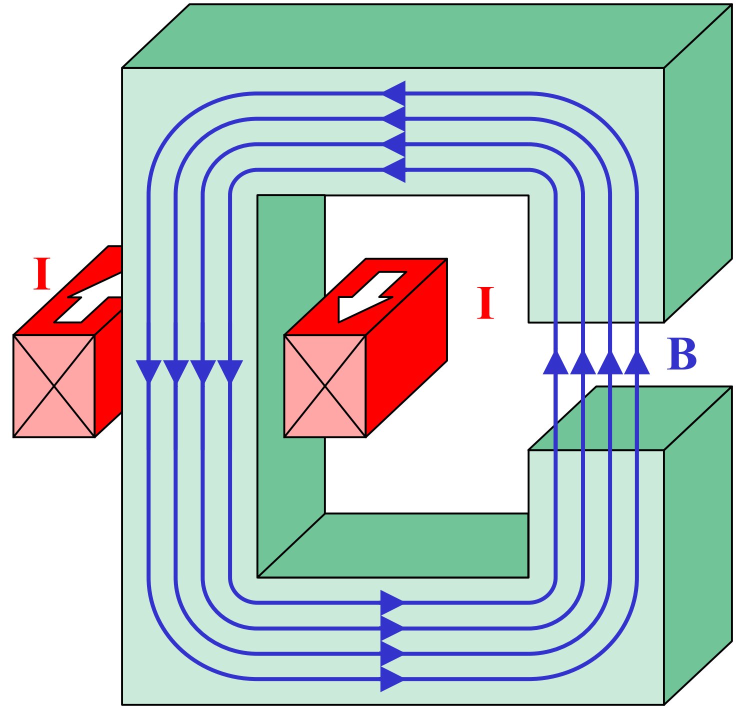
- the critical state model & screening currents
- flux jumping

Fields and ways to create them: conventional

- conventional electromagnets have an iron yoke
  - reduces magnetic reluctance
  - reduces ampere turns required
  - reduces power consumption
- iron guides and shapes the field



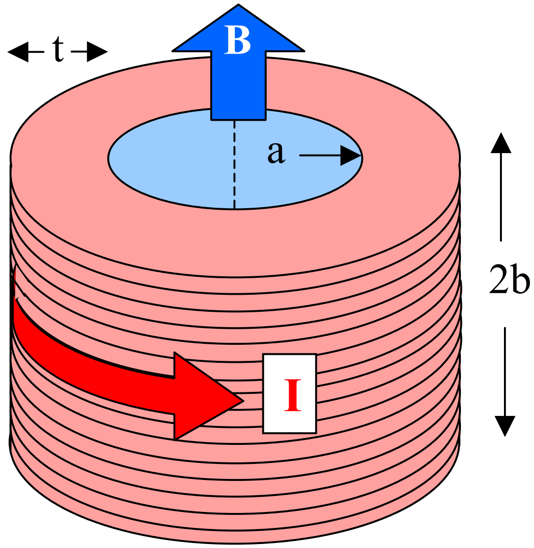
BUT iron saturates at ~ 2T



Iron electromagnet
– for accelerators, motors, transformers, generators etc

for higher fields we cannot rely on iron
field must be created and shaped by the winding

Solenoids



- no iron - field shape depends only on the winding
- azimuthal current flow, eg wire wound on bobbin, axial field
- the field produced by an infinitely long solenoid is

$$B = \mu_o NI = \mu_o J_e t$$

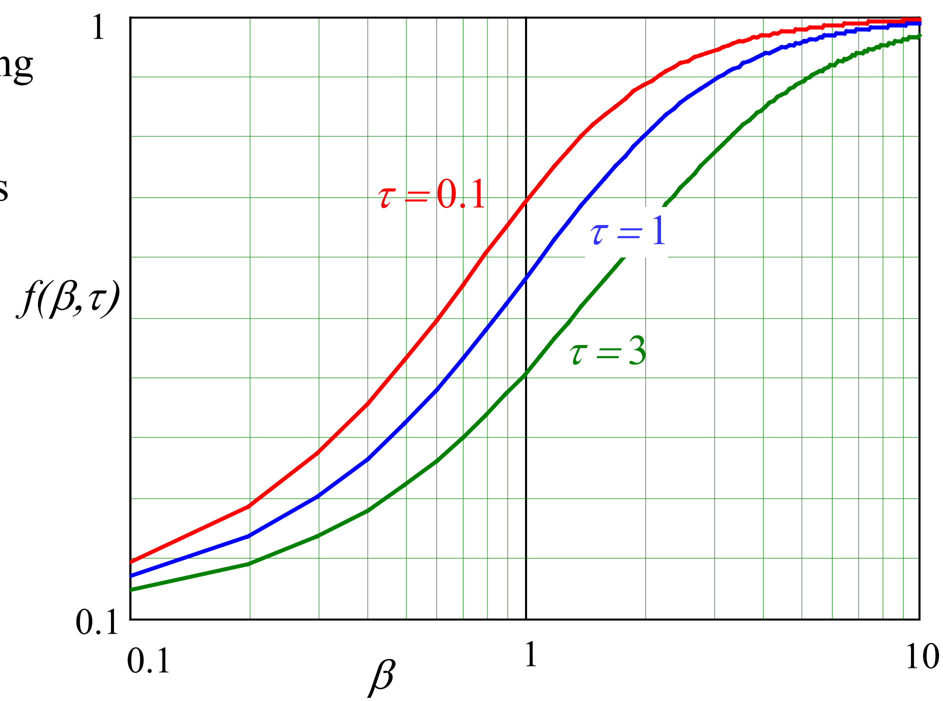
where N = number of turns/unit length, I = current , J_e= engineering current density

- so high $J_e \Rightarrow$ thin compact economical winding
- in solenoids of finite length the central field is

$$B = \mu_o J_e t f(\beta,\tau)$$

where $\beta = b/a \quad \tau = t/a$

- field uniformity and the ratio of peak field to central field get worse in short fat solenoids

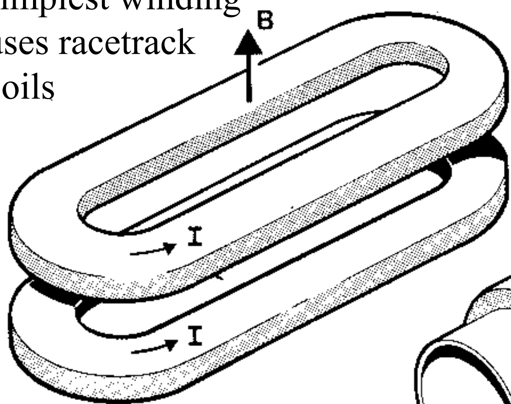


Superconducting solenoids

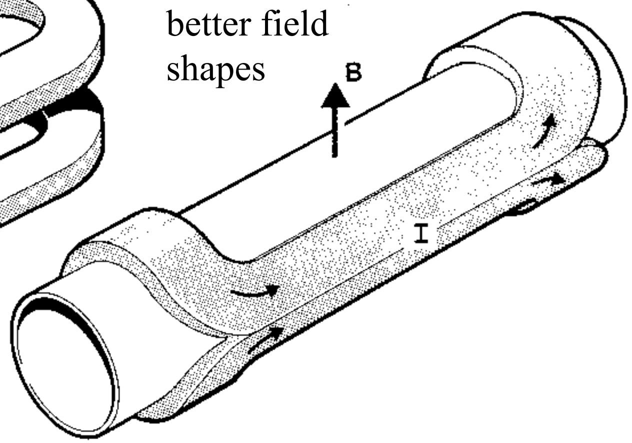
Superconducting solenoid for research

Accelerators need transverse fields

simplest winding uses racetrack coils

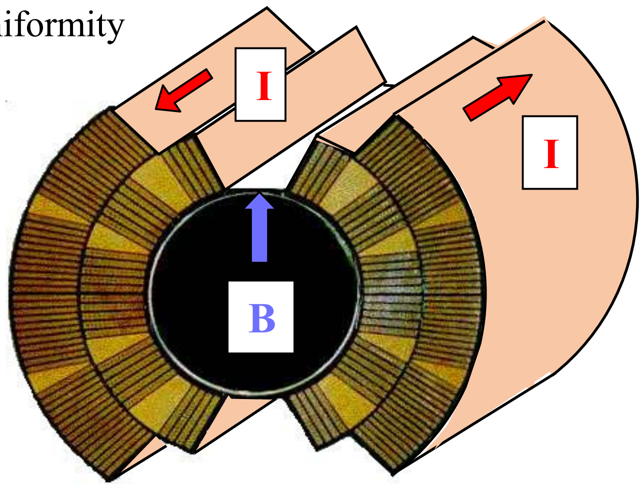


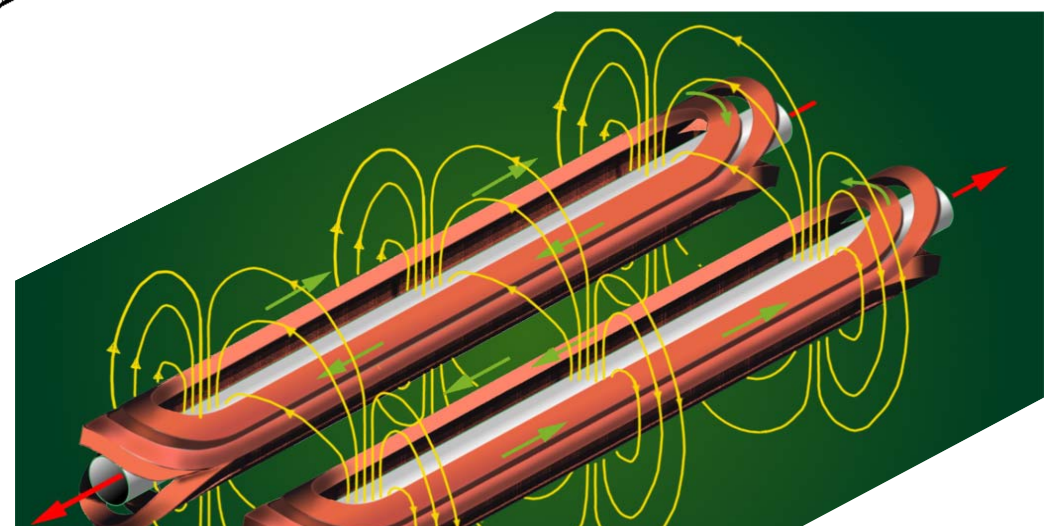
'saddle' coils make better field shapes



- some iron - but field shape is set mainly by the winding
- used when the long dimension is transverse to the field, eg accelerator magnets
- known as ***dipole*** magnets (because the iron version has 2 poles)

special winding cross sections for good uniformity



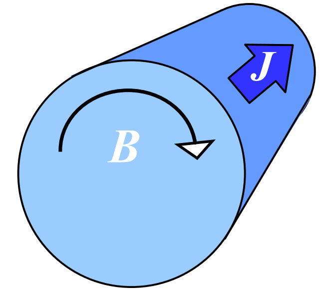


LHC has 'up' & 'down' dipoles side by side

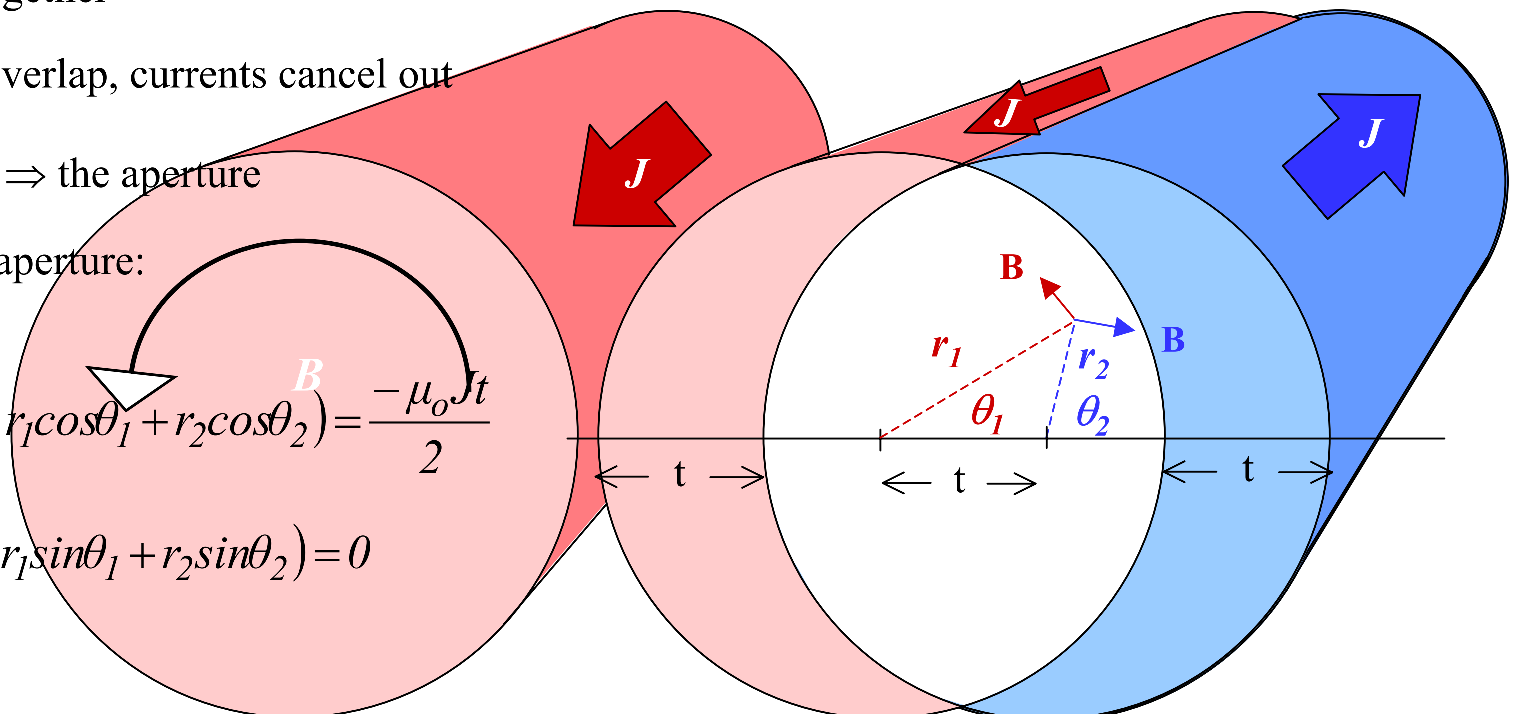
Dipole field from overlapping cylinders

Ampere's law for the field inside a cylinder carrying uniform current density

$$\oint B.ds = 2\pi r B = \mu_o I = \mu_o \pi r^2 J \qquad B = \frac{\mu_o J r}{2}$$



- two cylinders with opposite currents
- push them together
- where they overlap, currents cancel out
- zero current $\Rightarrow$ the aperture
- fields in the aperture:



$$B_y = \frac{\mu_o J}{2}\left(-r_1 cos\theta_1 + r_2 cos\theta_2\right) = \frac{-\mu_o J t}{2}$$

$$B_x = \frac{\mu_o J}{2}\left(-r_1 sin\theta_1 + r_2 sin\theta_2\right) = 0$$

- thus the two overlapping cylinders give a perfect dipole field

$$B_y = \frac{-\mu_o J_e t}{2}$$

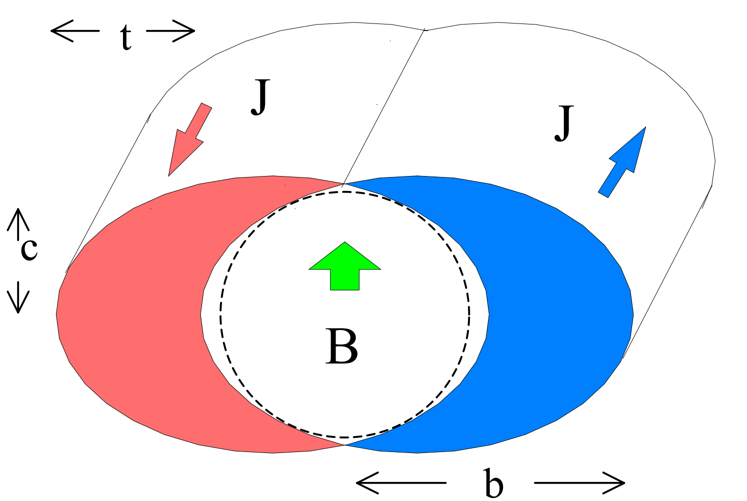
Overlapping ellipses

The same trick also works with overlapping ellipses carrying uniform current density .

Analysis is much more complex,

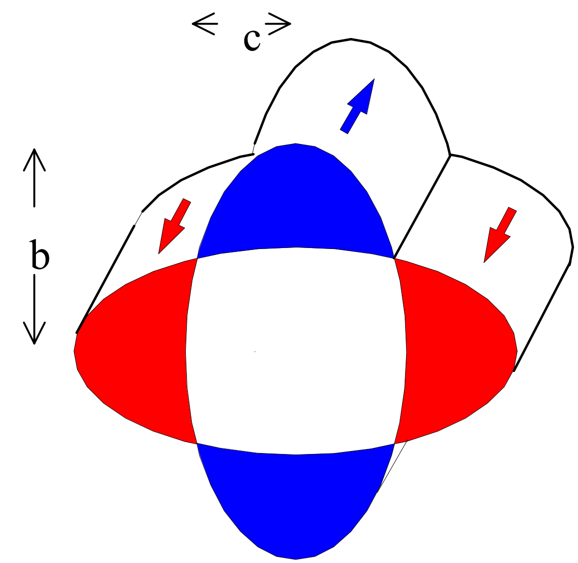
see eg Beth *J Appl Phys Vol 38 pp 4689*

superpose two ellipses carrying opposite J



$$B_y = \mu_o J\, t\, \frac{c}{b+c} \qquad B_x = 0$$

Or if we turn one ellipse through 90°



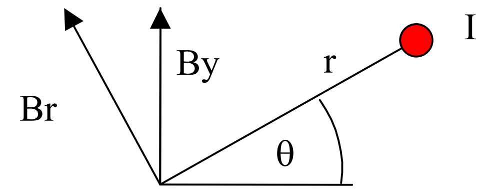
$$B_y = \mu_o J \frac{b-c}{b+c} x \qquad Bx = \mu_o J \frac{b-c}{b+c} y$$

A perfect quadrupole!

Efficiency of producing a dipole field

red filament carrying current I produces field B_y at the origin

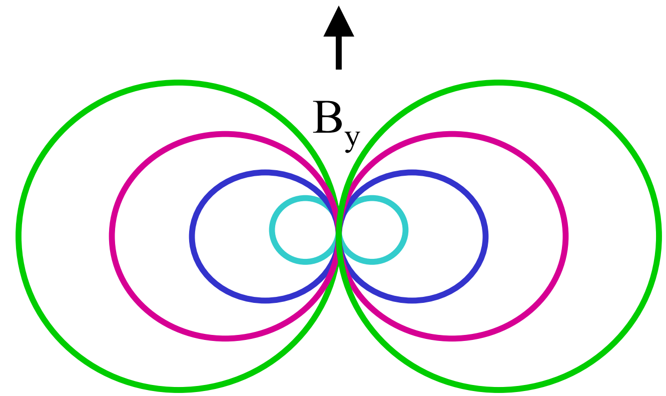
$$B_y = \mu_o \frac{I}{2\pi r} Cos(\theta)$$

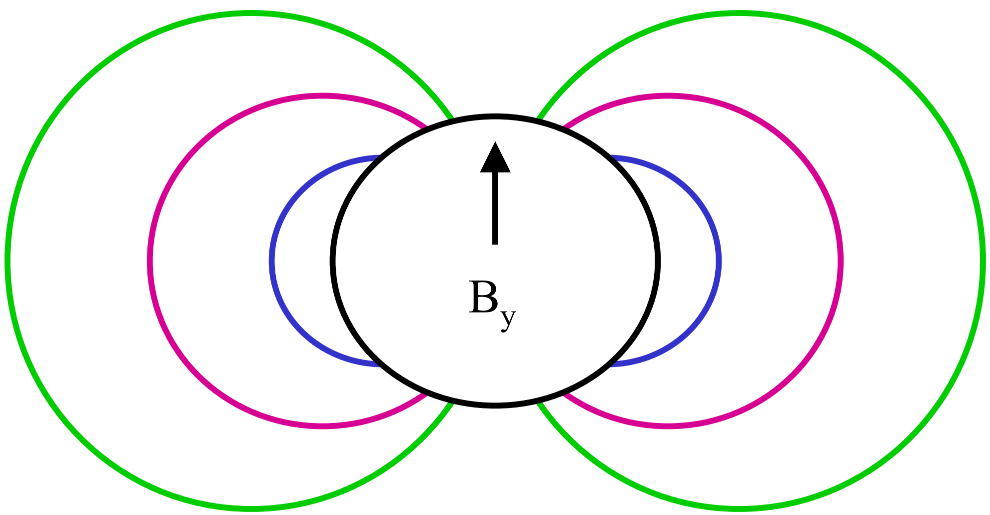


- efficiency = *B / I*
- contours of constant efficiency

$$r = d\,Cos(\theta)$$

defines circles of diameter *d* touching the origin





- finally cut out the aperture
- these are the 'iso-efficiency' contours of current to produce field in the aperture
- blue contour is most efficient, green is least

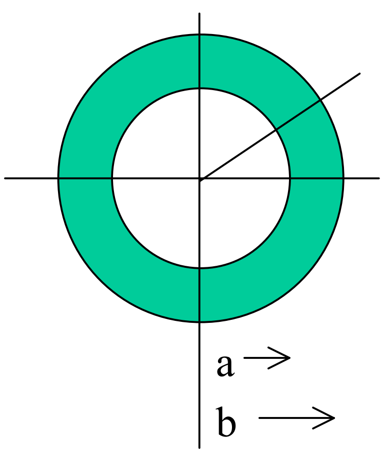
Windings of distributed current density

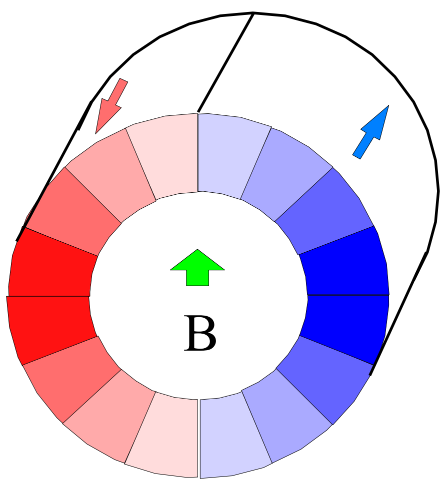
Analyse thin current sheets flowing on the surface of a cylinder using complex algebra. Let the ***linear*** current density ***(Amps per m of circumference)*** be $g_n = g_o \cos(n\theta)$ ***(Am^{-1})***

For $n = 1$ we find a pure dipole field inside the cylinder, $n = 2$ gives a quadrupole etc.
Now superpose many cylinders of increasing radius to get a thick walled cylinder carrying an (area) current density ***(Am^{-2})*** $J_n = J_o \cos(n\theta)$

n=1 $J_1(\theta) = J_0 \, \delta r \cos\theta$

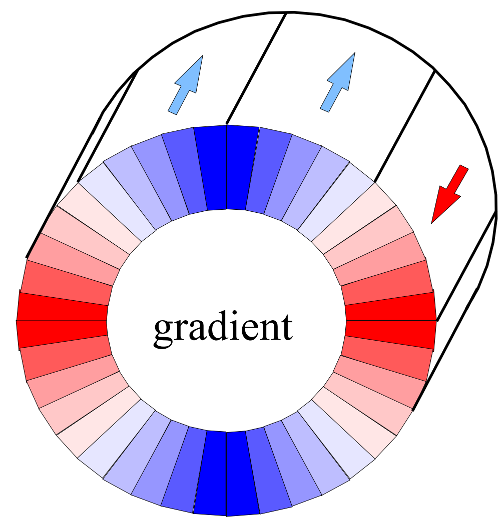
$$B_x = 0 \qquad B_y = -\mu_o J_0 (b-a)/2$$



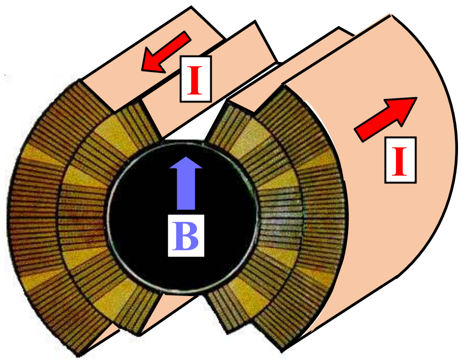


n=2 $J_2(\theta) = J_0 \cos 2\theta$

$$B_x = \frac{\mu_o J_0}{2} y \ln\left(\frac{b}{a}\right) \qquad B_y = \frac{\mu_o J_0}{2} x \ln\left(\frac{b}{a}\right)$$

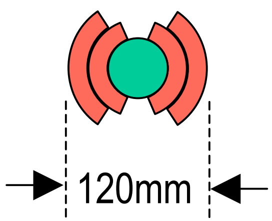


Importance of current density in dipoles



LHC dipole

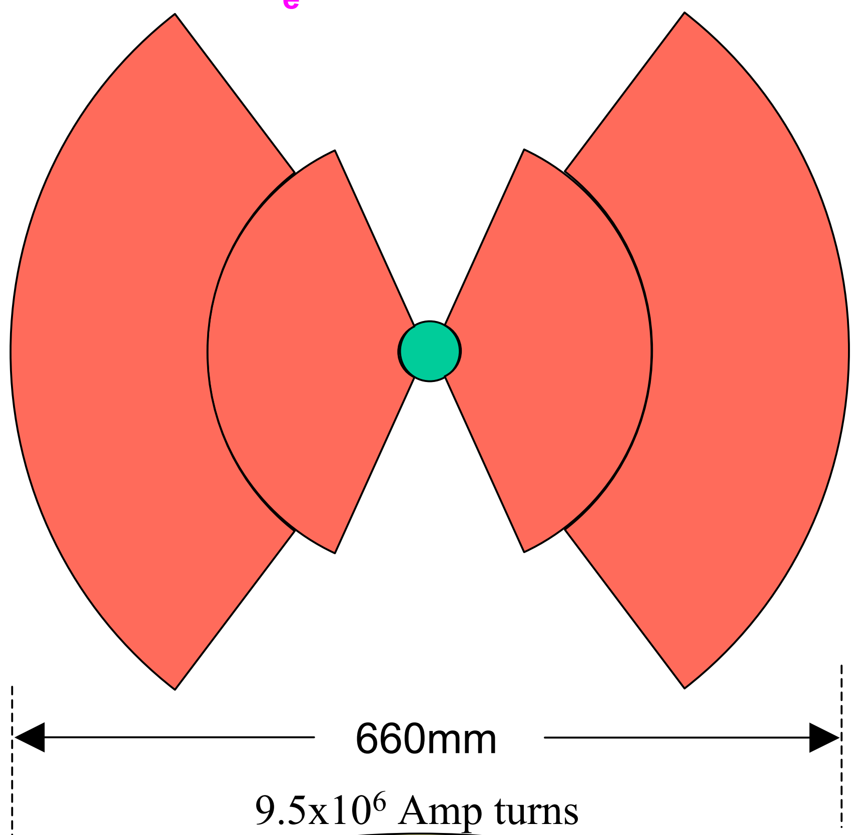
J_e = 375 Amm^{-2}



9.5x10^5 Amp turns

=1.9x10^6 A.m per m

J_e = 37.5 Amm^{-2}



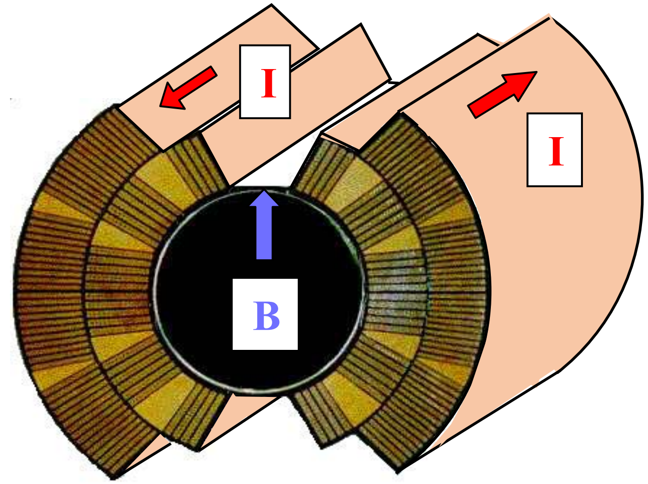
9.5x10^6 Amp turns

=1.9x10^7 A.m per m

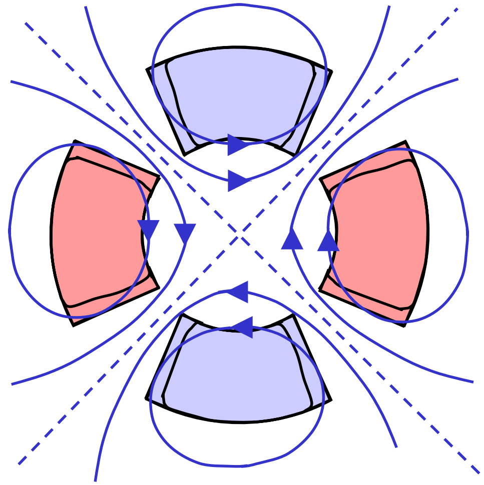
field produced by a perfect dipole is

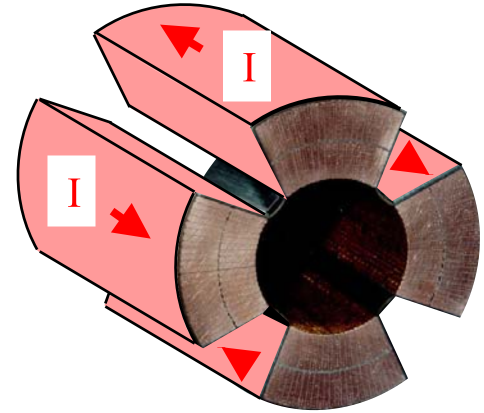
$$B = \mu_o J_e \frac{t}{2}$$

Dipole Magnets

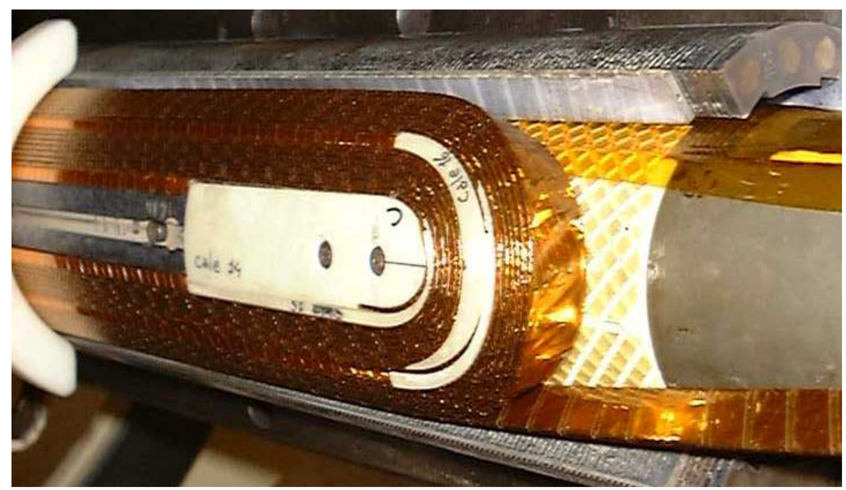


Quadrupole windings

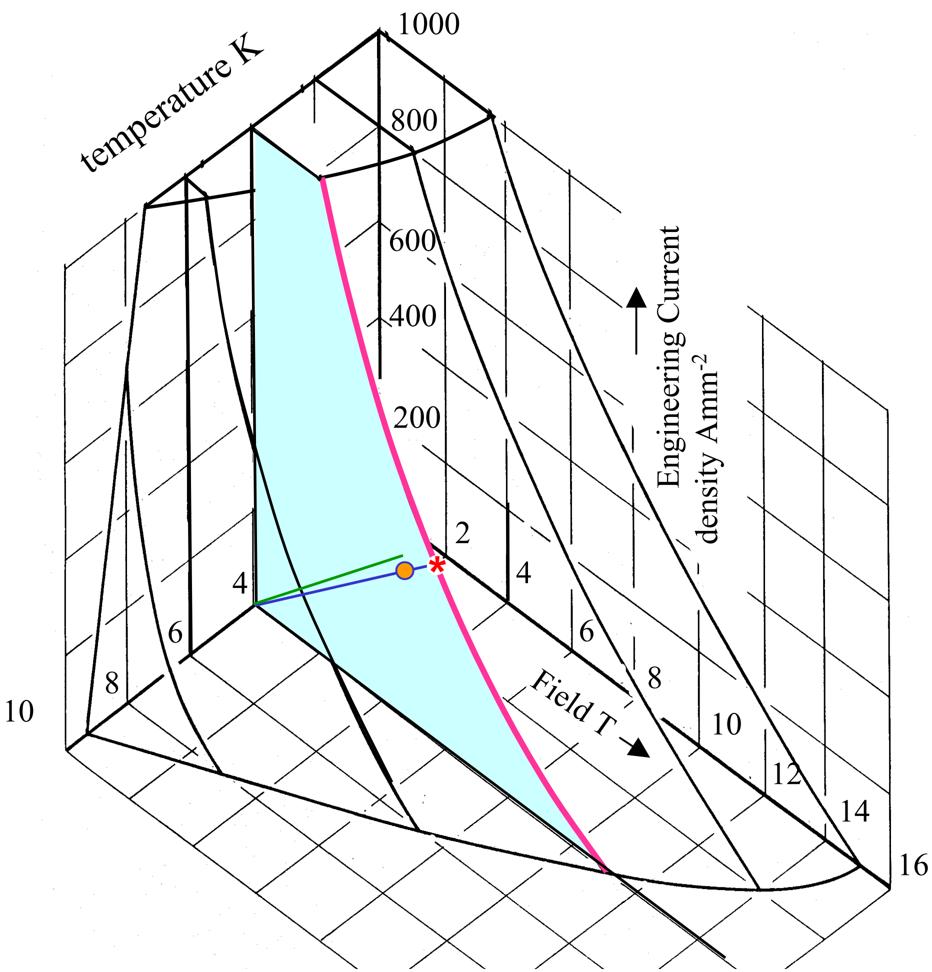


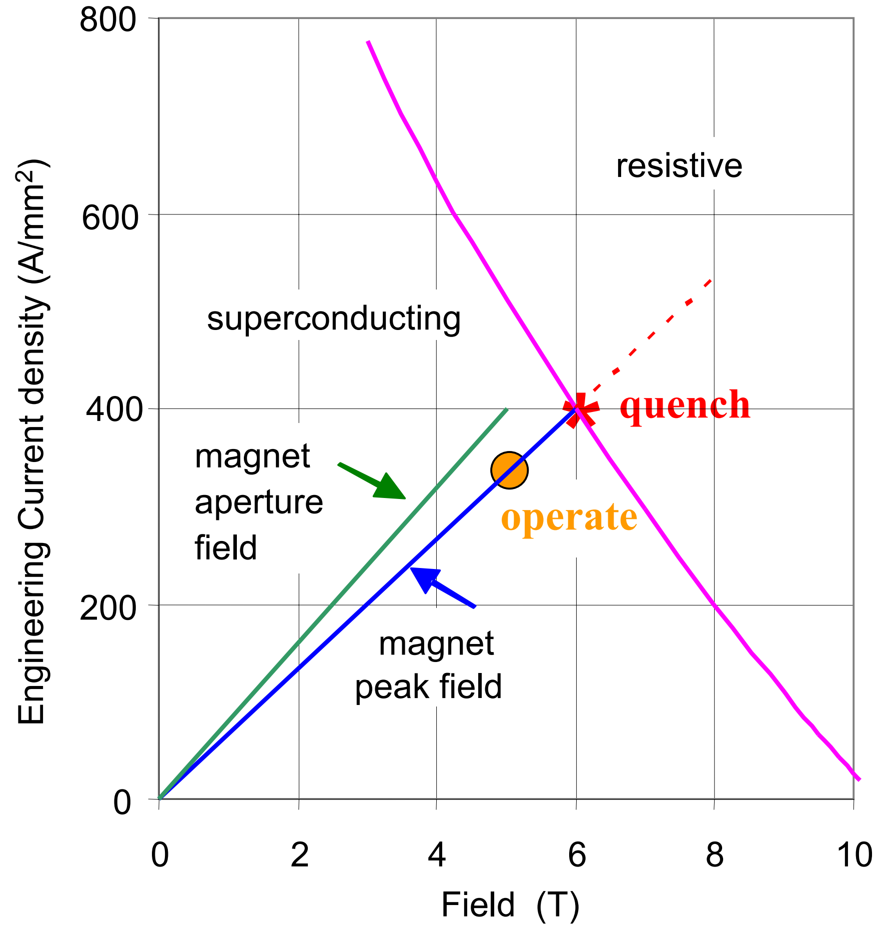


$B_x = ky$ $\qquad$ $B_y = kx$



Critical line and magnet load lines

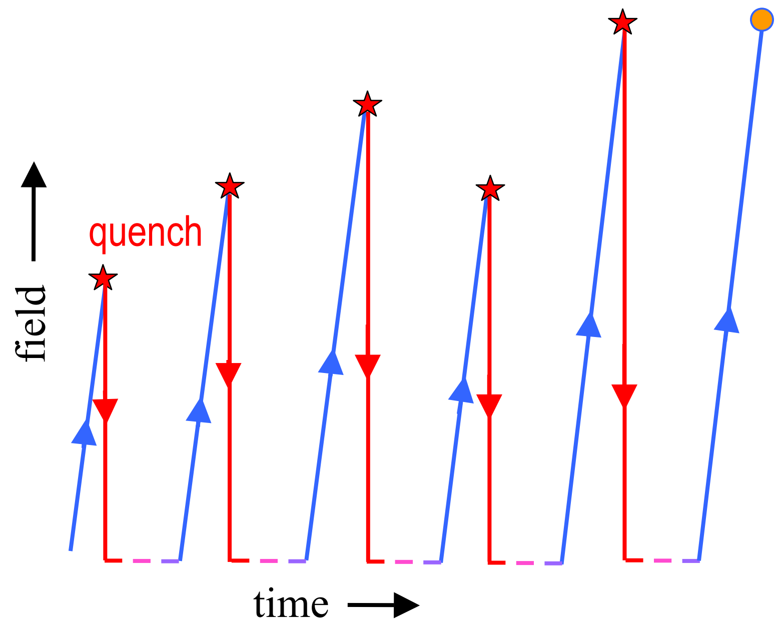


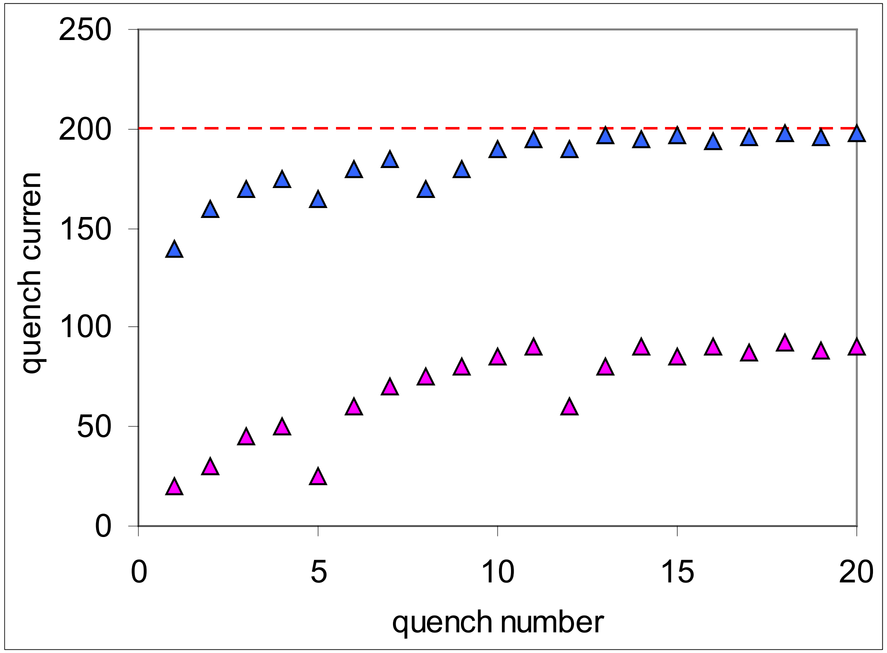


we expect the magnet to go resistive *'quench'* where the peak field load line crosses the critical current line *
usually back off from this extreme point and operate at ●

Degraded performance and 'training' of magnets

- an early disappointment for magnet makers came when the current (and field) of a magnet was ramped up for the first time
- instead of going up to the critical line, it 'quenched' (went resistive) at less than the expected current
- at the next try it did better
- known as ***training***

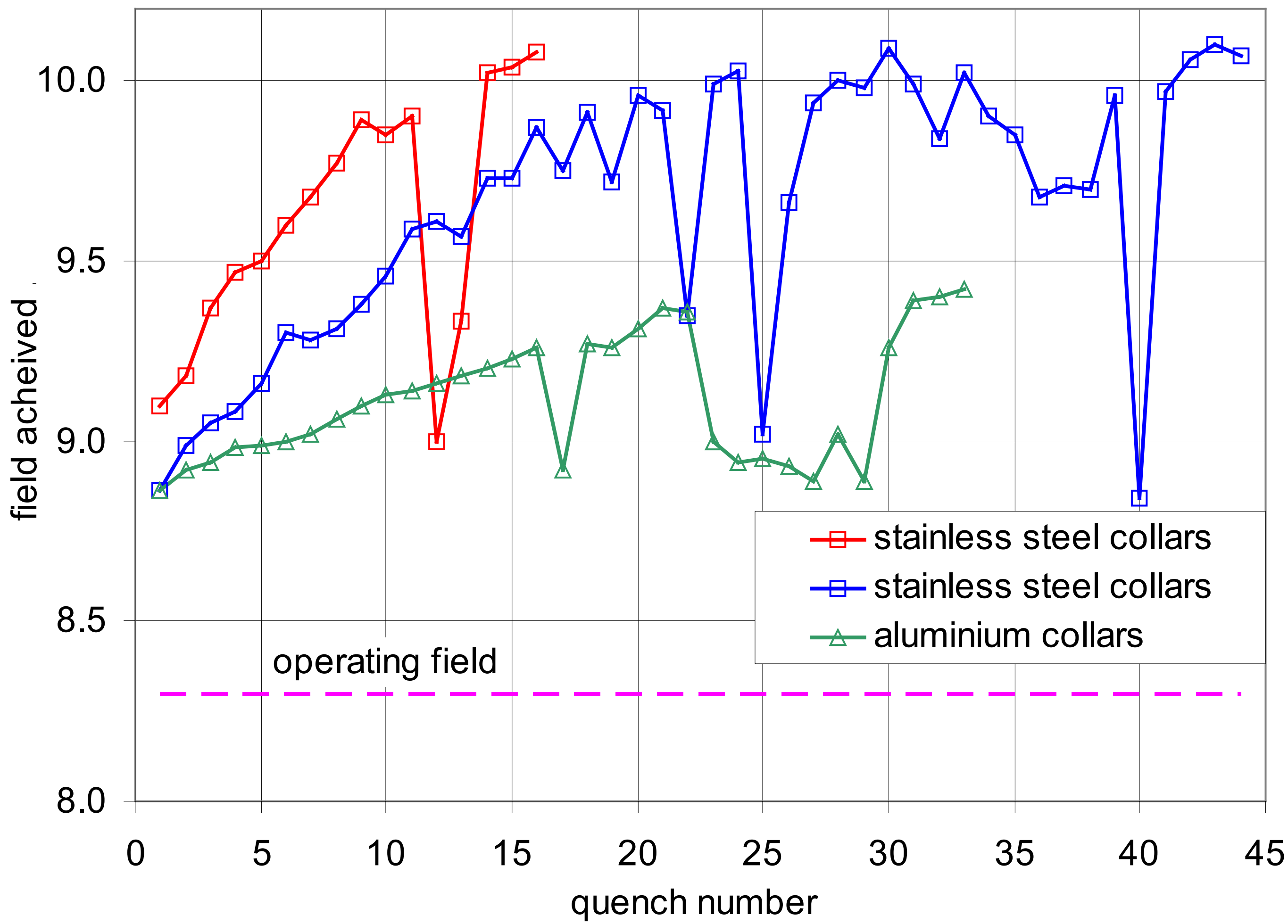




- after a ***quench***, the stored energy of the magnet is dissipated in the magnet, raising its temperature way above critical
- you must wait for it to cool down and then try again
- well made magnets ▲ are better than poorly made ▲

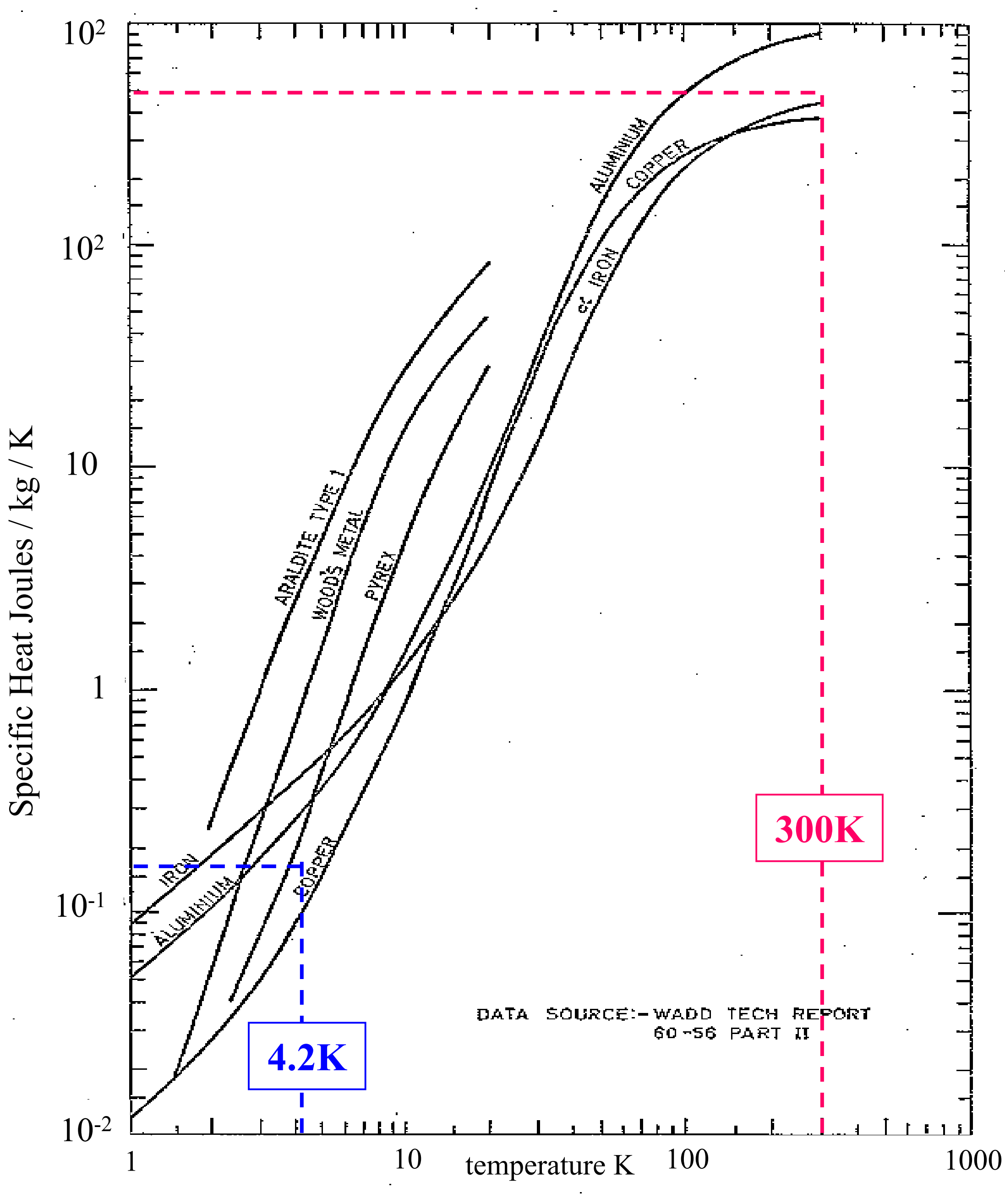
'Training' of magnets

- it's better than the old days, but training is still with us
- it seems to be affected by the construction technique of the magnet
- it can be wiped out if the magnet is warmed to room temperature
- 'de-training is the most worrysome feature



Training of LHC short prototype dipoles *(from A. Siemko)*

Causes of training: (1) low specific heat

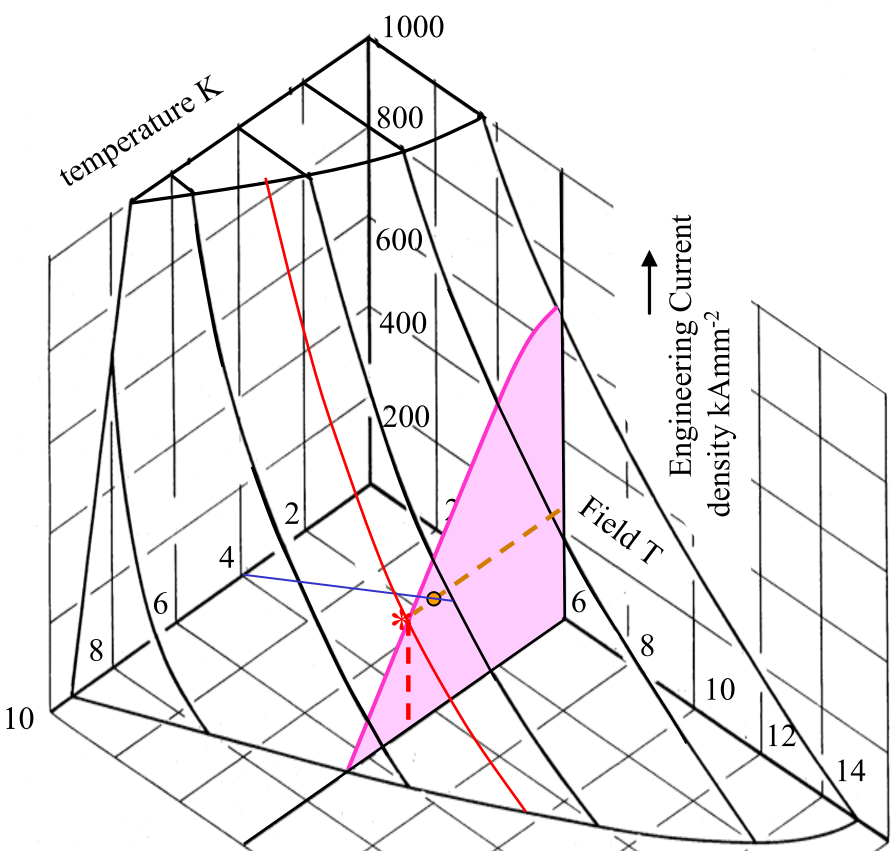


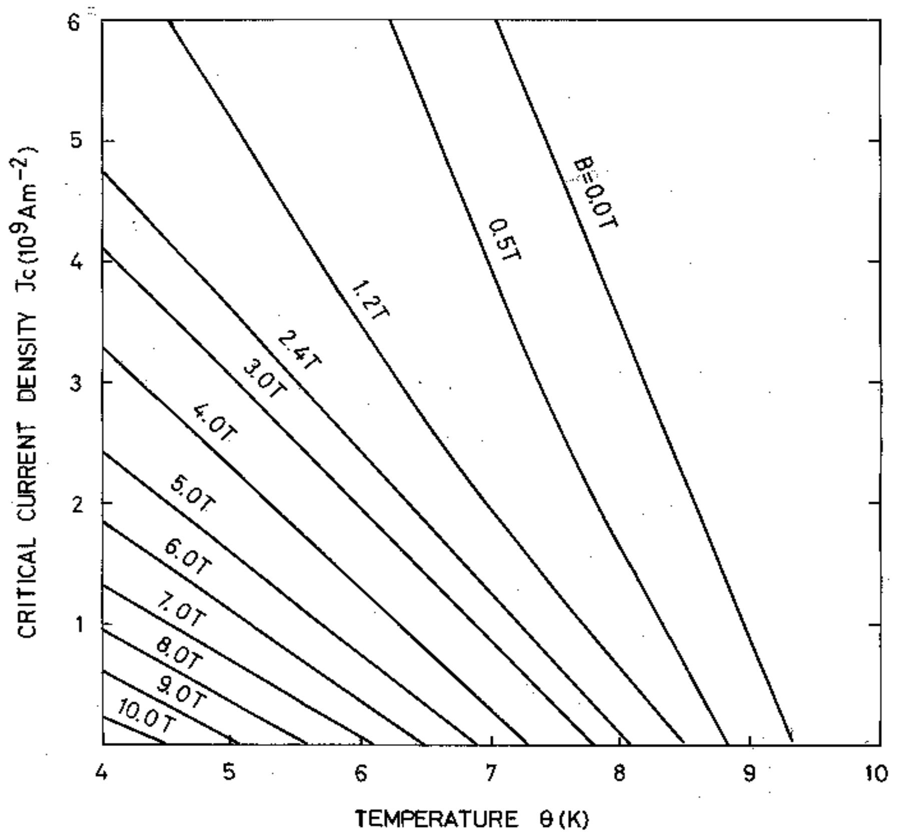
- the specific heat of all substances falls with temperature
- at 4.2K, it is ~2,000 times less than at room temperature
- a given release of energy within the winding thus produce a temperature rise 2,000 times greater than at room temperature
- the smallest energy release can therefore produce catastrophic effects

Causes of training: (2) J_c decreases with temperature

at any given field, the critical current of NbTi falls almost linearly with temperature

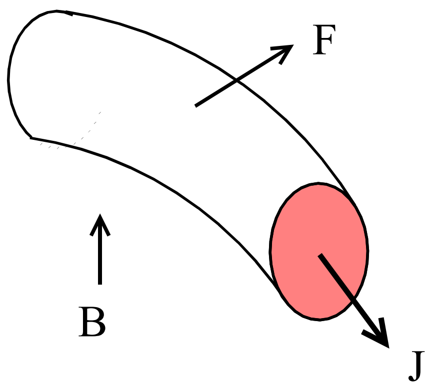
- so any temperature rise drives the conductor into the resistive state





Causes of training: (3) conductor motion

Conductors in a magnet are pushed by the electromagnetic forces. Sometimes they move suddenly under this force - the magnet 'creaks' as the stress comes on. A large fraction of the work done by the magnetic field in pushing the conductor is released as frictional heating



work done per unit length of conductor if it is pushed a distance δz

$$W = F.\delta z = B.I.\delta z$$

frictional heating per unit volume

$$Q = B.J.\delta z$$

typical numbers for NbTi:

B = 5T $J_{eng} = 5 \times 10^{8}$ $A.m^{-2}$

so if $\delta = 10$ μm

then $Q = 2.5 \times 10^{4}$ $J.m^{-3}$

Starting from 4.2K $\theta_{final} = 7.5K$

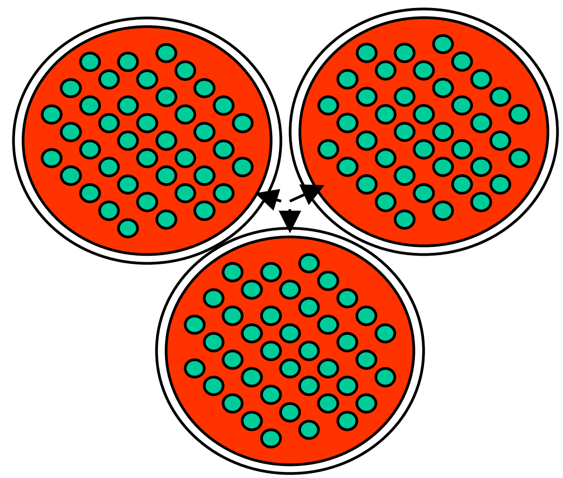
*can **you** engineer a winding to better than*

10 μm?

Causes of training: (4) resin cracking

We try to stop wire movement by impregnating the winding with epoxy resin. Unfortunately the resin contracts much more than the metal, so it goes into tension. Furthermore, almost all organic materials become brittle at low temperature. *brittleness + tension ⇒ cracking ⇒ energy release*

Calculate the stain energy induced in resin by differential thermal contraction



let: σ = tensile stress Y = Young's modulus
ε = differential strain ν = Poisson's ratio

typically: $\varepsilon = (11.5 - 3) \times 10^{-3}$ $Y = 7 \times 10^9$ Pa $\nu = {}^1/_3$

uniaxial strain $$Q_1 = \frac{\sigma^2}{2Y} = \frac{Y\varepsilon^2}{2}$$ $Q_1 = 2.5 \times 10^5$ J.m^{-3} $\theta_{final} = 16$K

triaxial strain $$Q_3 = \frac{3\sigma^2(1-2\nu)}{2Y} = \frac{3Y\varepsilon^2}{2(1-2\nu)}$$ $Q_3 = 2.3 \times 10^6$ J.m^{-3} $\theta_{final} = 28$K

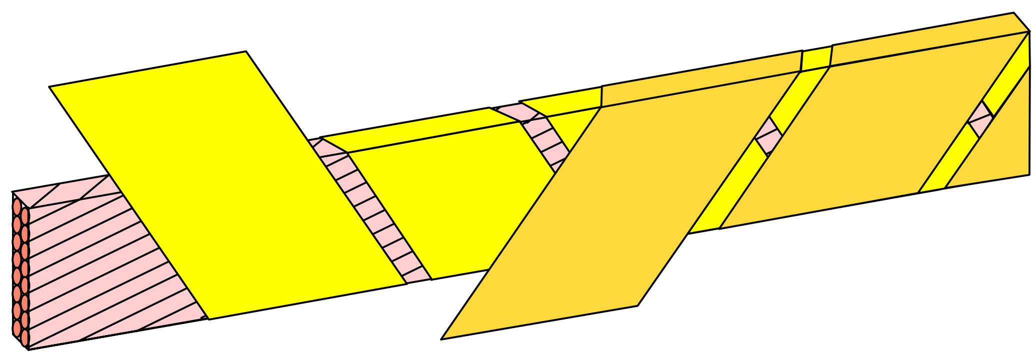
an unknown, but large, fraction of this stored energy will be released as heat during a crack

Interesting fact: magnets impregnated with paraffin wax show almost no training although the wax is full of cracks after cooldown.
Presumably the wax breaks at low σ before it has had chance to store up any strain energy

How to reduce training?

1) Reduce the disturbances occurring in the magnet winding

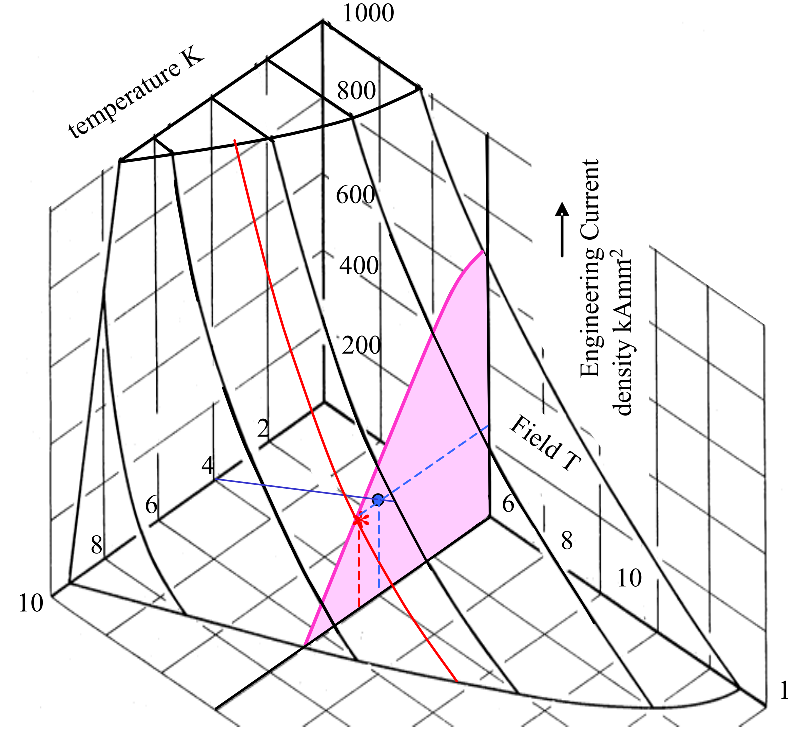
- make the winding fit together exactly to reduce movement of conductors under field forces
- pre-compress the winding to reduce movement under field forces
- if using resin, minimize the volume and choose a crack resistant type
- match thermal contractions, eg fill epoxy with mineral or glass fibre
- impregnate with wax - but poor mechanical properties
- most accelerator magnets are insulated using a Kapton film with a very thin adhesive coating on the outer face
  - away from the superconductor
- allows liquid helium to penetrate the cable

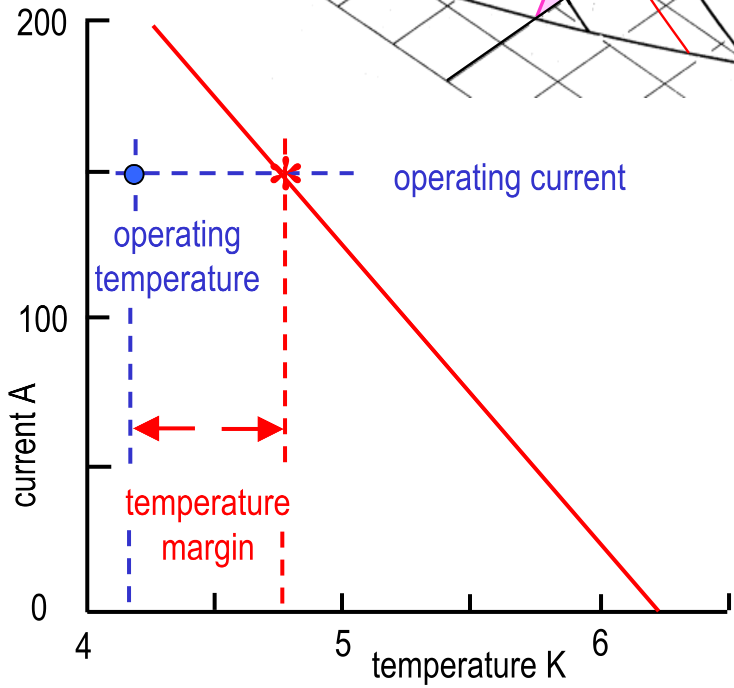


How to reduce training?

2) Make the conductor able to withstand disturbances without quenching

- increase the temperature margin
    - operate at lower current
    - higher critical temperature - HTS?
- increase the cooling
    - more cooled surface
    - better heat transfer
    - superfluid helium
- increase the specific heat
    - experiments with Gd_2O_2S $HoCu_2$ etc





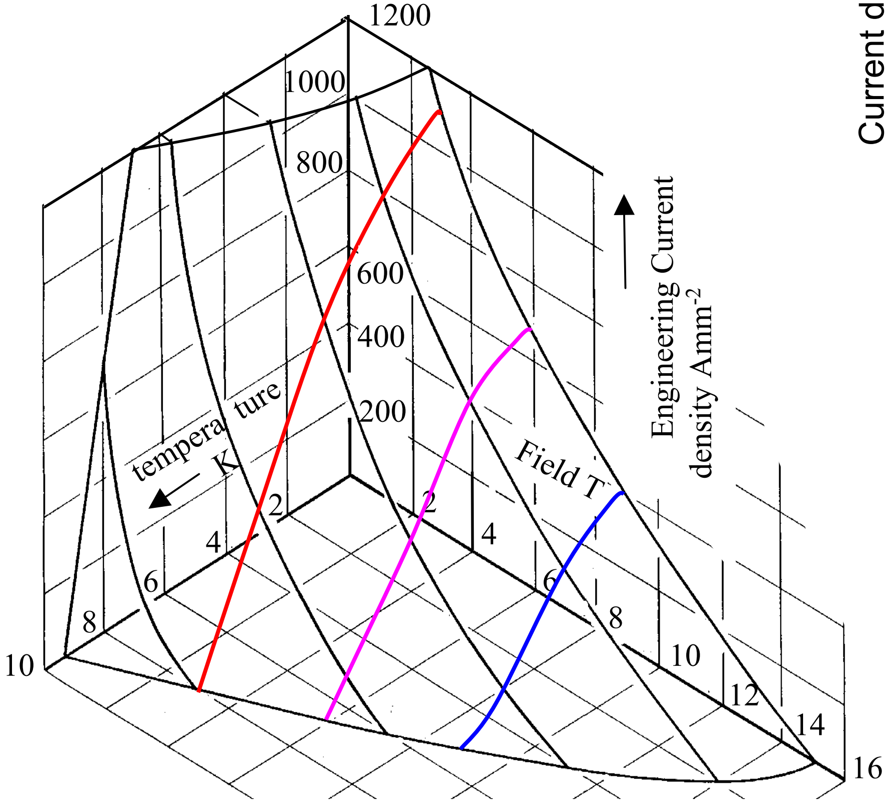
most of this may be characterized by a single number

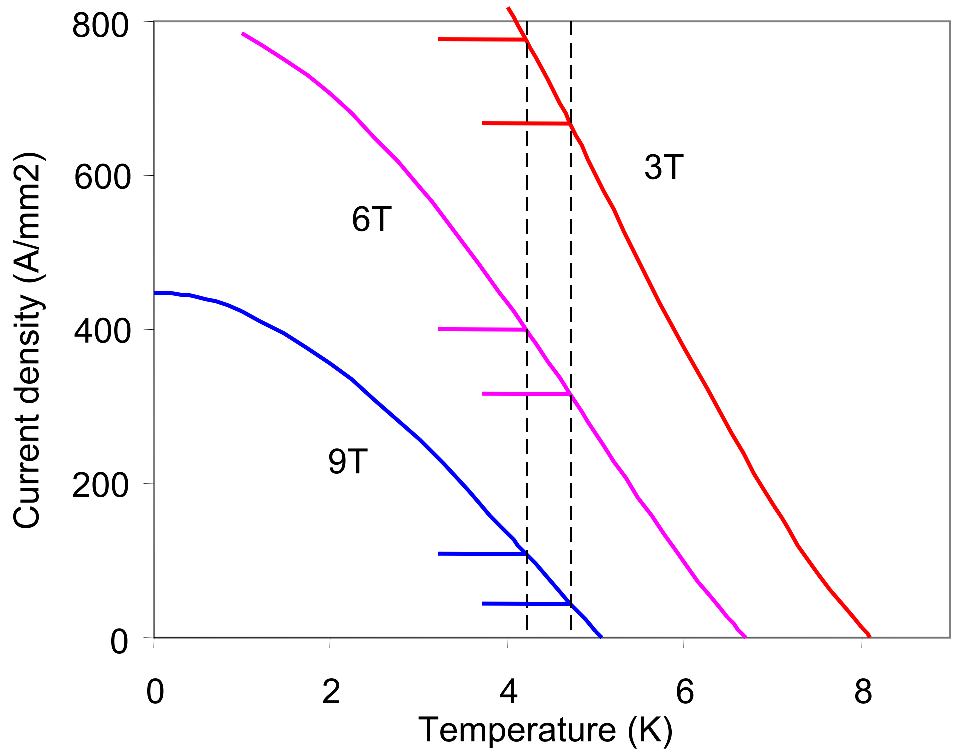
Minimum Quench Energy MQE

= energy input at a point which is just enough to trigger a quench

Temperature margin

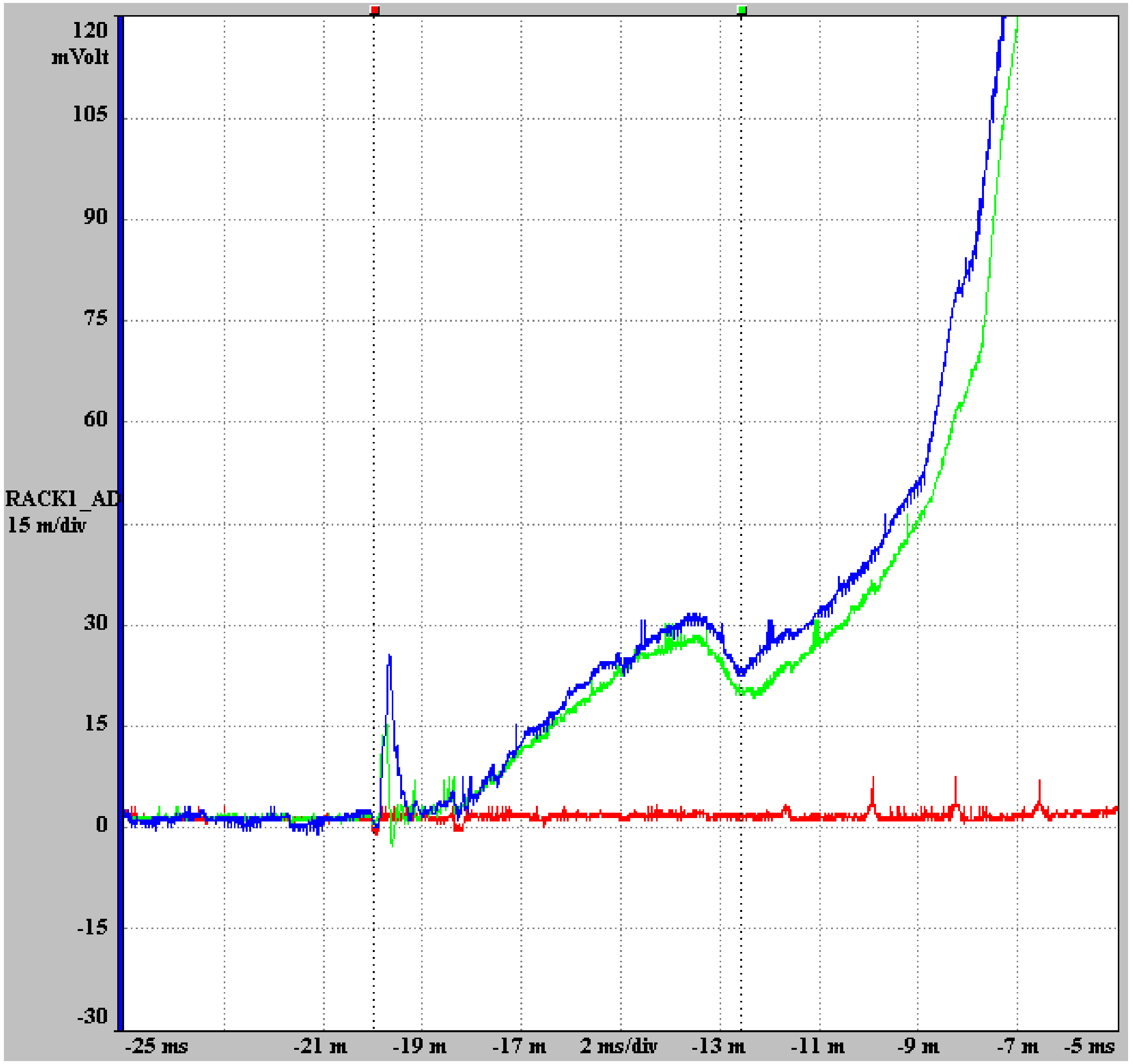
for a chosen temperature margin
- say 0.5K - the % reduction in
critical current gets worse at high field





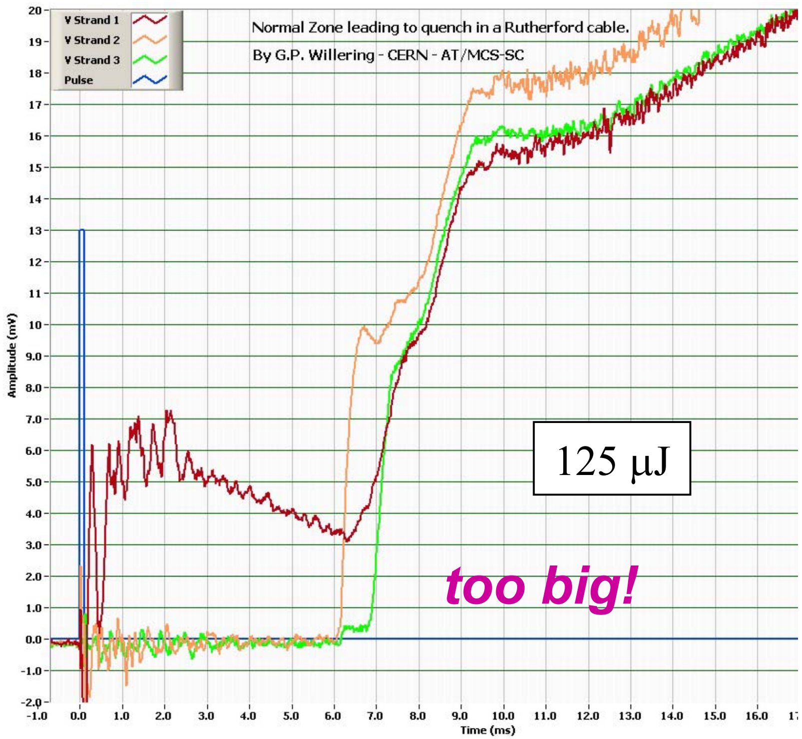
B (T)	J_c (4.2) Amm^{-2}	J_c(4.7) Amm^{-2}	J_c(4.7) / J_c(4.2)
3	3881	3335	**86%**
6	2000	1581	**79%**
9	545	227	**42%**

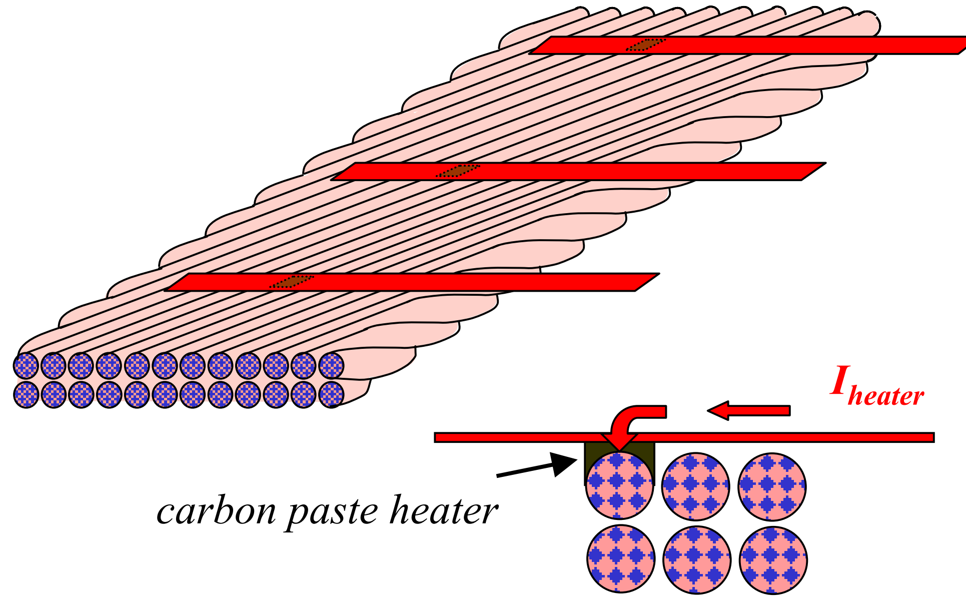
Quench initiation by a disturbance

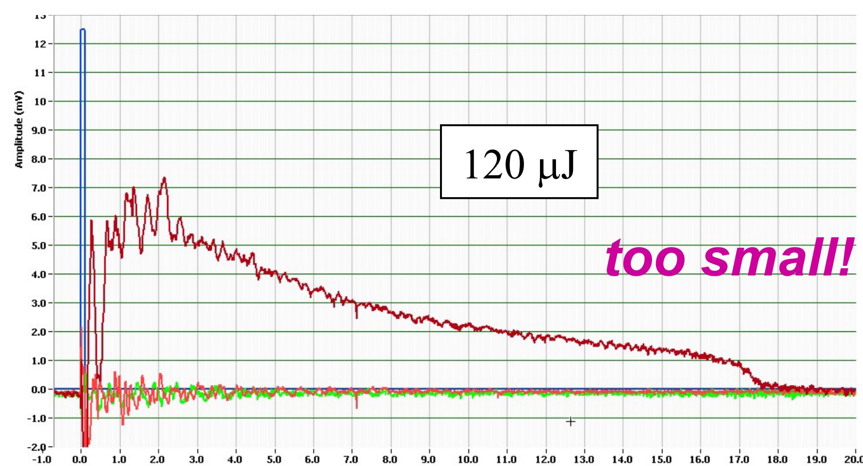


- CERN picture of the internal voltage in an LHC dipole just before a quench
- note the initiating spike - conductor motion?
- after the spike, conductor goes resistive, then it almost recovers
- but then goes on to a full quench
- can we design conductors that recover from a disturbance and thus avoid quenching the magnet?

Measuring the energy needed to quench a cable



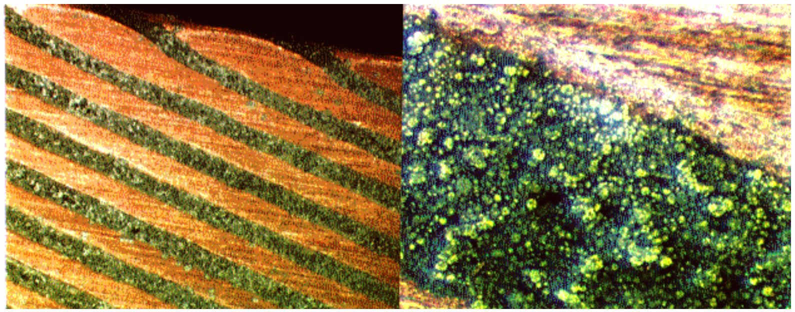




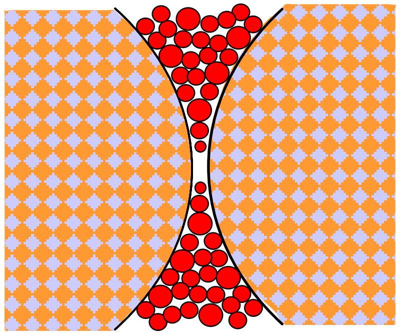
- pass a small pulse of current from the copper foil to the superconducting wire
- generates heat in the carbon paste contact
- how much to quench the cable?
- find the *Minimum Quench Energy MQE*

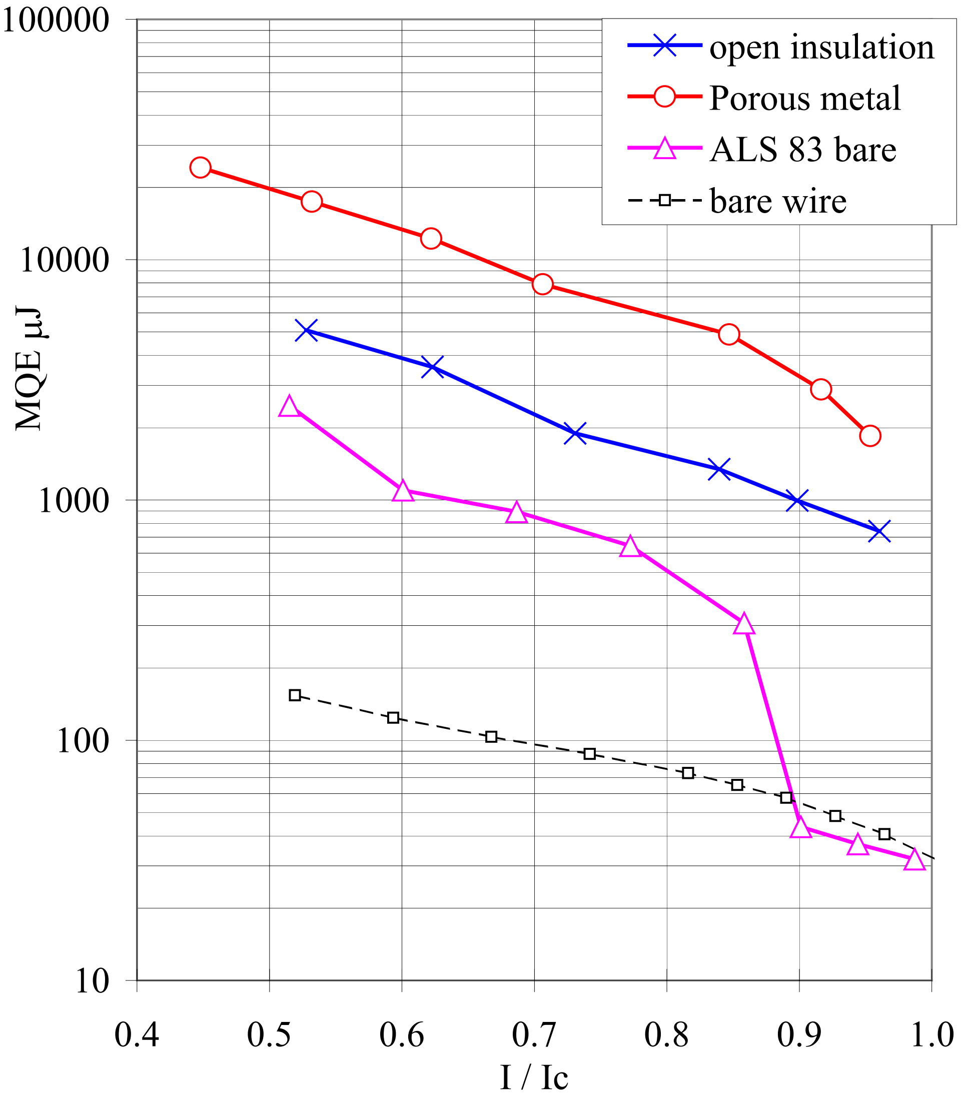
Different cables have different MQEs

- similar cables with different cooling
- better cooling gives higher MQE
- high MQE is best because it is harder to quench the magnet

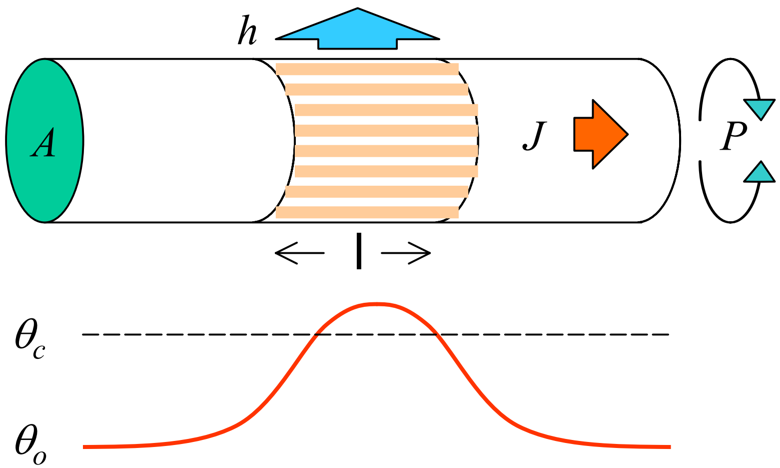


- experimental cable with porous metal heat exchanger
- excellent heat transfer to the liquid helium coolant





Factors affecting the Minimum Quench Energy



- think of a conductor where a short section has been heated, so that it is resistive
- if heat is conducted out of the resistive zone faster than it is generated, the zone will shrink - vice versa it will grow.
- the boundary between these two conditions is called the *minimum propagating zone MPZ*
- for best stability make MPZ as large as possible

the balance point may be found by equating heat generation to heat removed. *Very* approximately, we have:

$$\frac{2kA(\theta_c-\theta_o)}{l}+hPl(\theta_c-\theta_o)=J_c^2\rho Al$$

$$l=\left\{\frac{2k(\theta_c-\theta_o)}{J_c^2\rho-\frac{hP}{A}(\theta_c-\theta_o)}\right\}^{\frac{1}{2}}$$

where: k = thermal conductivity ρ = resistivity A = cross sectional area of conductor

h = heat transfer coefficient to coolant – if there is any in contact

P = cooled perimeter of conductor

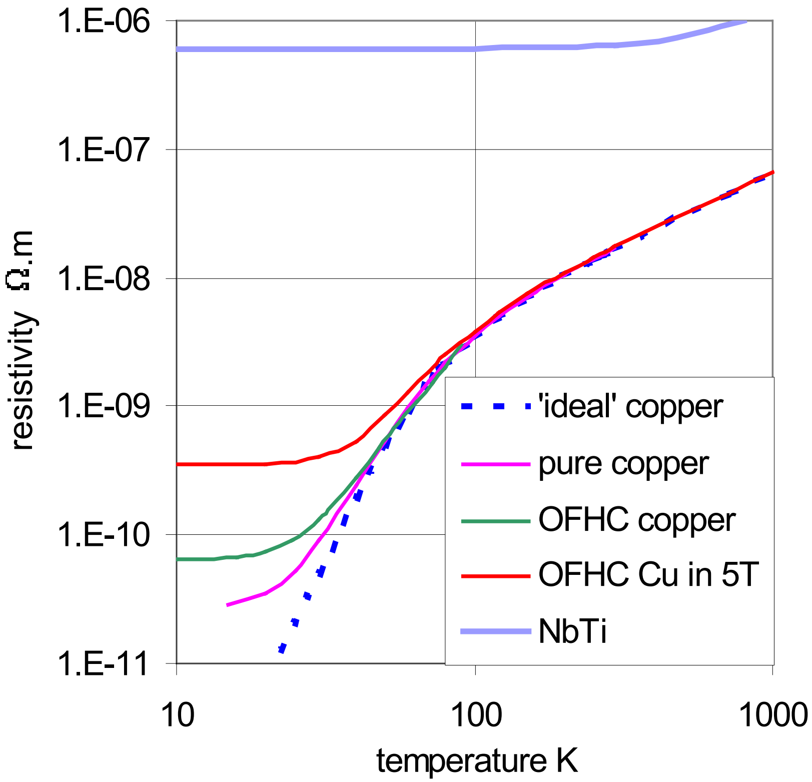
Energy to set up MPZ is the Minimum Quench Energy

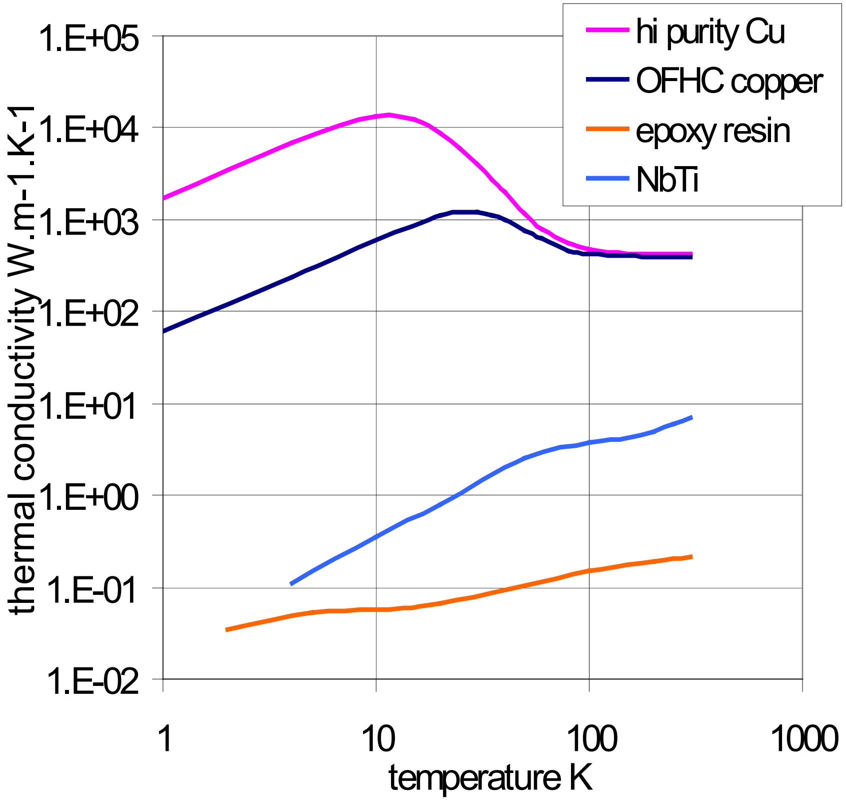
long MPZ ⇒ large MQE

How to make a long MPZ ⇒ large MQE

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \frac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

- make thermal conductivity k large
- make resistivity ρ small
- make heat transfer *hP/A* large (but ⇒ low J_{eng})



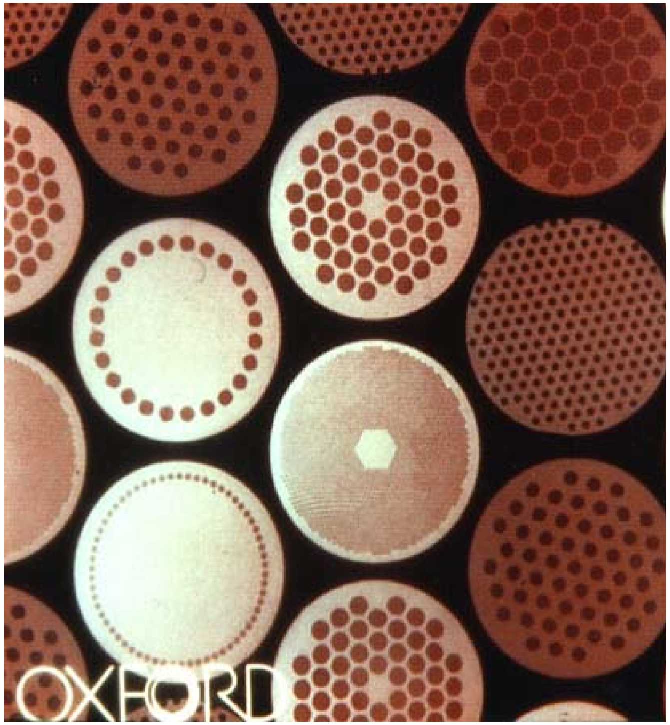


Large MPZ ⇒ large MQE ⇒ less training

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \frac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

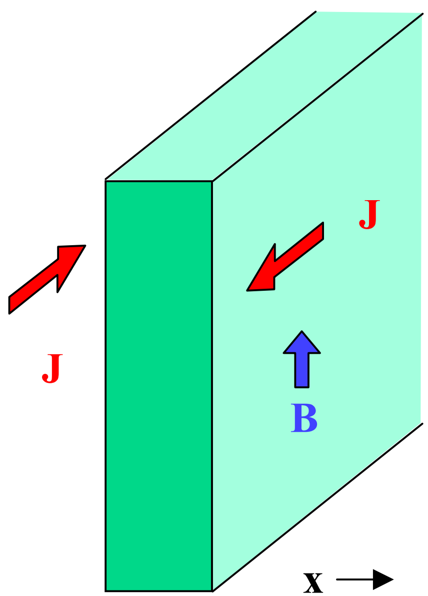
- make thermal conductivity *k* large
- make resistivity ρ small
- make heat transfer term *hP/A* large

- NbTi has high ρ and low *k*
- copper has low ρ and high *k*
- mix copper and NbTi in a filamentary composite wire
- make NbTi in *fine filaments* for intimate mixing
- maximum diameter of filaments ~ 50μm
- make the windings porous to liquid helium
  - superfluid is best
- fine filaments also eliminate flux jumping
  (see later slides)



Another cause of training: flux jumping

- when a superconductor is subjected to a changing magnetic field, screening currents are induced to flow
- *screening currents* are in addition to the *transport current*, which comes from the power supply
- they are like eddy currents but, because there is no resistance, they don't decay



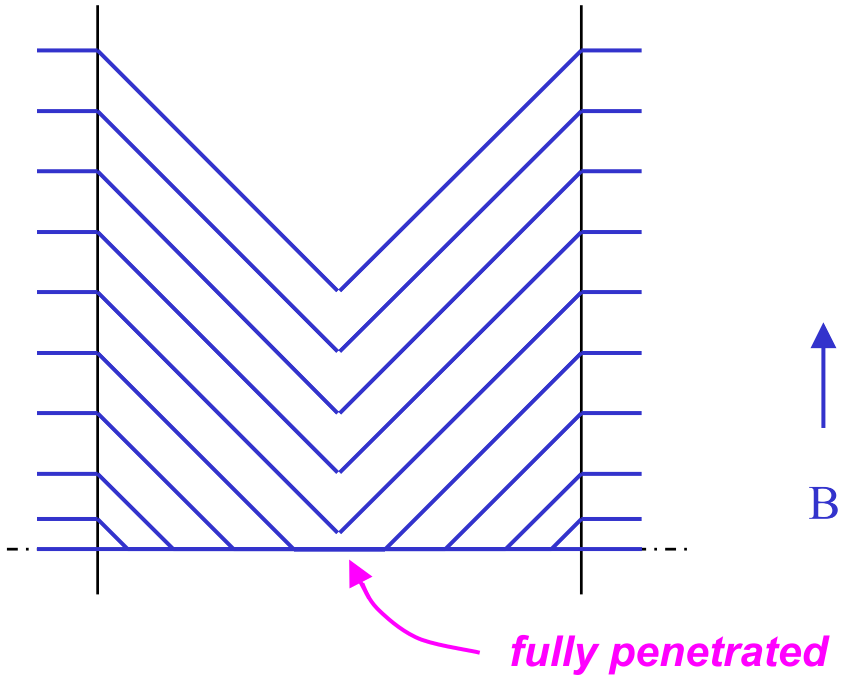
- usual model is a superconducting slab in a changing magnetic field B_y
- assume it's infinitely long in the z and y directions - simplifies to a 1 dim problem
- *dB/dt* induces an electric field E which causes screening currents to flow at critical current density J_c
- known as the *critical state model* or *Bean model*
- in the 1 dim infinite slab geometry, Maxwell's equation says

$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

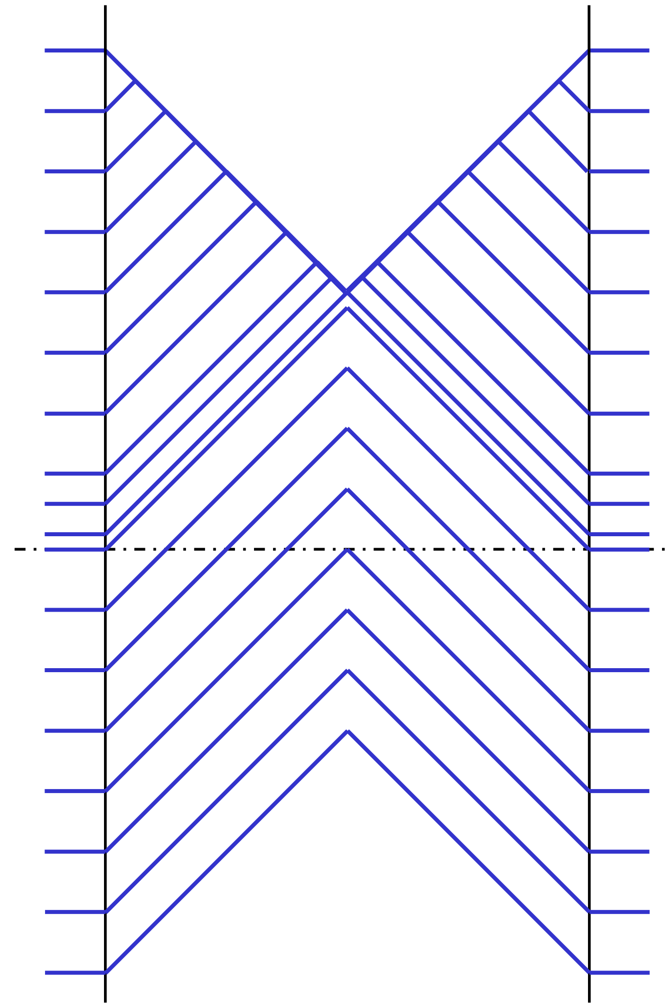
- so uniform J_c means a constant field gradient inside the superconductor

The flux penetration process

plot field profile across the slab



field increasing from zero



field decreasing through zero

Bean critical state model

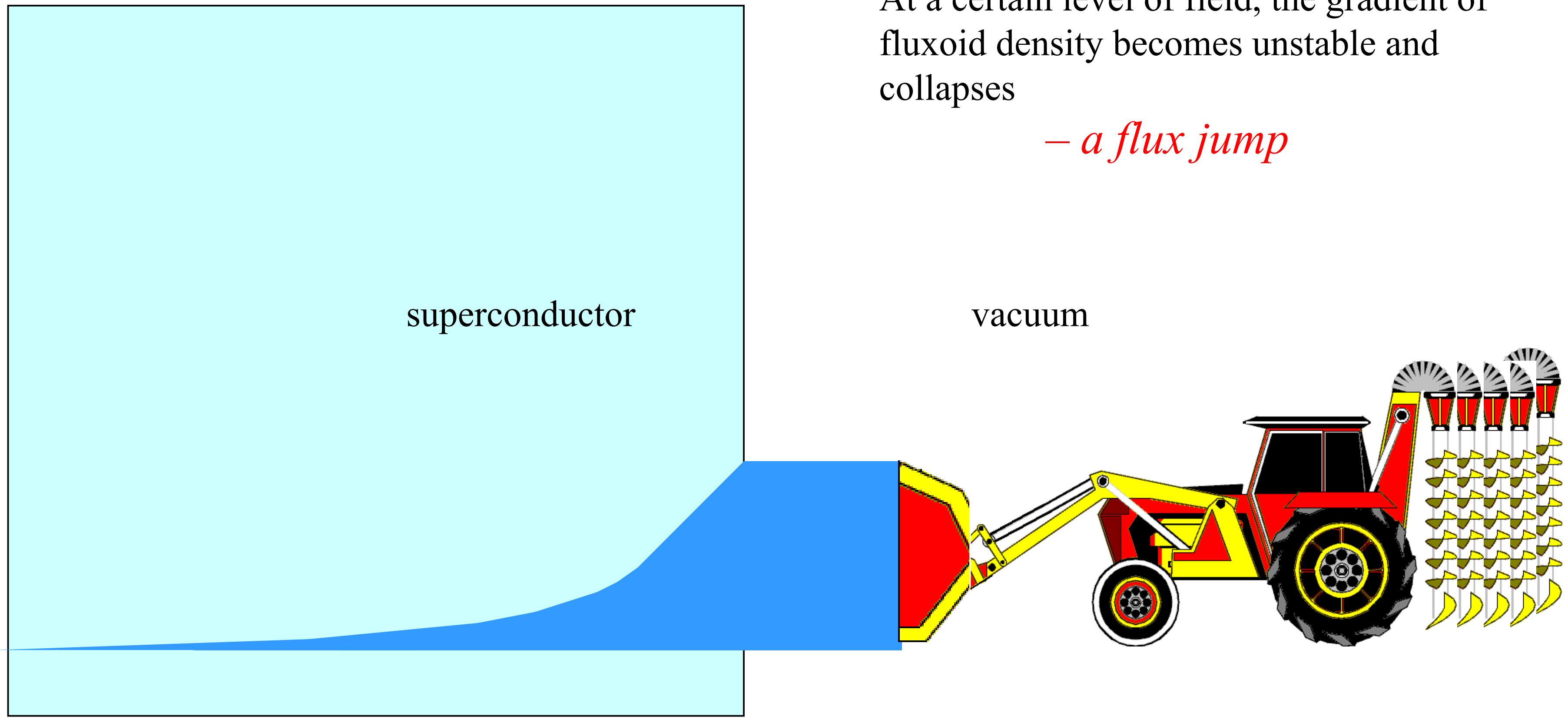
- **current density everywhere is $\pm J_c$ or zero**
- **change comes in from the outer surface**

Flux penetration from another viewpoint

Think of the screening currents, in terms of a gradient in fluxoid density within the superconductor. Pressure from the increasing external field pushes the fluxoids against the pinning force, and causes them to penetrate, with a characteristic gradient in fluxoid density

At a certain level of field, the gradient of fluxoid density becomes unstable and collapses

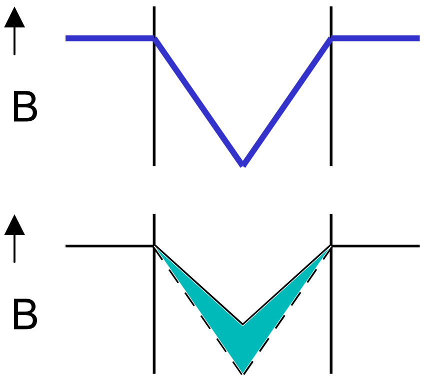
– a flux jump



Flux jumping: why it happens

Unstable behaviour is shown by all type 2 and HT superconductors when subjected to a magnetic field

It arises because:-



magnetic field induces screening currents, flowing at critical density J_c

*** reduction in screening currents allows flux to move into the superconductor**

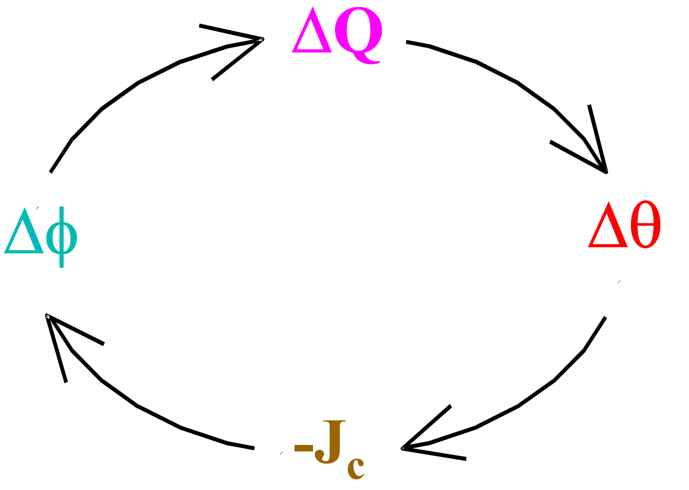
flux motion dissipates energy

thermal diffusivity in superconductors is low, so energy dissipation causes local temperature rise

critical current density falls with increasing temperature

go to *

positive feedback
⇒ avalanche



Cure flux jumping by making superconductor in the form of fine filaments – weakens $\Delta J_c \Rightarrow \Delta\phi \Rightarrow \Delta Q$

Flux jumping: the numbers for NbTi

criterion for stability against flux jumping
a = half width of filament

$$a = \frac{1}{J_c}\left\{\frac{3\gamma\, C(\theta_c - \theta_o)}{\mu_o}\right\}^{\frac{1}{2}}$$

typical figures for NbTi at 4.2K and 1T

J_c *critical current density* = $7.5 \times 10^{9}\ Am^{-2}$

γ *density* = $6.2 \times 10^{3}\ kg.m^{3}$

C *specific heat* = $0.89\ J.kg^{-1}K^{-1}$

θ_c *critical temperature* = 9.0K

so a = 33mm, ie 66mm diameter filaments

Notes:

- least stable at low field because J_c is highest
- instability gets worse with decreasing temperature because J_c increases and C decreases
- criterion gives the size at which filament is just stable against infinitely small disturbances
  - still sensitive to moderate disturbances, eg mechanical movement
- better to go somewhat smaller than the limiting size
- in practice 50µm diameter seems to work OK

Flux jumping is a solved problem✓

Concluding remarks

- superconducting magnets can make higher fields than conventional because they don't need iron which saturates at 2T - although iron is often used for shielding
- to get different field shapes you have to shape the winding (not the iron)
- practical winding shapes are derived from the theoretical ideal overlapping ellipses and $J = J_o Cos\theta$
- engineering current density is important for a compact economic magnet design
- expected magnet performance is given by the intersection of the load line and critical surface
- degraded performance and training are still a problem for magnets - and de-training is worse
- improve training by good winding construction
  $\Rightarrow$ no movement, low thermal contraction, no cracking
- improve training by making the conductor have a high MQE
  - requires NbTi to be in good contact with copper
  $\Rightarrow$ fine filaments
- changing fields induce screening currents in all superconductors $\Rightarrow$ flux jumping
- flux jumping used to cause degraded magnet performance but fine filaments now cure it